

Let X_i be the result of a single roll. X_i will have one of the values in the set $(.7, .8, .9, 1, 1.2, 1.5)$ with equal probability. We wish to see the probability distribution of the multiplicative outcome O_N , which is the result of letting all the winning ride for N consecutive games. That is,

$$O_N = \prod_{i=1}^N X_i$$

Using bonanova's brilliant insight, we examine the log of both sides instead

$$\begin{aligned} \log(O_N) &= \log\left(\prod_{i=1}^N X_i\right) \\ &= \sum_{i=1}^N \log(X_i) \end{aligned}$$

So, we simply have to examine the distribution of $\log(O_N) = \sum_{i=1}^N \log(X_i)$. Fortunately, this is fairly straightforward using the fundamental statistical fact (central limit theorem) that the sum of independently distributed variables will have a gaussian distribution.

Let's look at the distribution for $\log(X_i)$, which will have one of the values in the set $(\log(.7), \log(.8), \log(.9), \log(1), \log(1.2), \log(1.5))$ with equal probability. $\log(X_i)$ has a mean of -0.01623206 and a standard deviation of 0.2530392. From the central limit theorem, we see that

$$\log(O_N) \sim \text{Gaussian}(-0.01623206 \cdot N, \sqrt{N} \cdot 0.2530392),$$

In other words, $\log(O_N)$ is normally distributed with mean $-0.01623206 \cdot N$ and standard deviation $\sqrt{N} \cdot 0.2530392$.

EXAMPLE: let $N = 1000$. That is, we play the game 1000 times and bet the entire bankroll each time. From above, $\log(O_N)$ has a mean of $-0.01623206 \cdot N = -16.23$ and a standard deviation of $\sqrt{N} \cdot 0.2530392 = 8.001$.

If we want to calculate the proportion of times that we end up winning (larger bankroll at the end), then we simply have to calculate $P(\log(O_{1000}) > 0)$. Given mean -16.23 and sd 8.765, the z-score for that case is $(0 - (-16.23)) / 8.765 = 2.03$.

Using the pdf of the gaussian curve, it is easy to see from the z-score that $P(\log(O_{1000}) > 0) = .02125462$. That is, there is a 2.12% chance that we would end up with a larger bankroll at the end of 1000 rolls.

I simulated 10,000 trials wherein each trial is a simple realization of O_{1000} , that is, playing the game 1000 times and let the winning ride. The end result is that 215 of those trials end up with positive winning. $215/10000 = .0215$, which is quite close to the expected value of .02125462