

Multiplying those 7s by 11 comes close, but then the first digit is 8.

Dividing 100 7-s by 7 we get 100 1-s. The number we seek is going to be divisible by 7 and the number represented by 100 1-s. Using long multiplication we can get the following structure:

11111111...11111 (one hundred 1-s) *

$x_1 x_2 x_3$

$x_1 x_2 x_3$

$x_1 x_2 x_3$

$x_1 x_2 x_3$

.....

$x_1 x_2 x_3$

$x_1 x_2 x_3$

$x_1 x_2 x_3$

xxx.....x

In this example I use three-digit number $x_1 x_2 x_3$.

From this it is apparent that x_3 , x_2+x_3 , $x_1+x_2+x_3$ +possible carry , x_1+x_2 +carry , and x_1 +carry must all end with 4, 5, or 7. On top of that the $x_1 x_2 x_3$ must be divisible by 7.

I see no such 2 or 3-digit numbers, no does a proof come to mind that such number with more than 3 digits does not exist.